PROBLEMS ON SOLVABLE FOR x

Prob [1] Solve $xp = 1+p^2$

Sol^n - $xp = 1+p^2$

$$x = \frac{1}{p} + p \quad \text{— (1)}$$

$$= f(p)$$

$\Rightarrow$ solvable for x

Diff. w.r.to y both sides

$$\frac{dx}{dy} = -\frac{1}{p^2}\frac{dp}{dy} + \frac{dp}{dy}$$

$$\frac{1}{p} = -\frac{1}{p^2}\frac{dp}{dy} + \frac{dp}{dy}$$

$$\frac{1}{p} = \left(-\frac{1}{p^2} + 1\right)\frac{dp}{dy}$$

$$dy = \left(-\frac{1}{p} + p\right)dp$$

Integ, $y = -\log p + \frac{p^2}{2} + c$ — (2)

The p-eliminant of (1) and (2) constitutes the solution.

Prob [2] Solve $xp^2 = 1+p^2$

Sol^n $xp^2 = 1+p^2$

$$x = \frac{1}{p^2} + 1 = f(p)$$

$\Rightarrow$ solvable for x

$$\therefore x = 1 + \frac{1}{p^2} \quad \text{— (1)}$$

Diff. w.r.to y both sides

$$\frac{dx}{dy} = 0 - \frac{2}{p^3}\frac{dp}{dy}$$

$$\frac{1}{p} = -\frac{2}{p^3}\frac{dp}{dy}$$

$$dy = -\frac{2}{p^2}dp$$

Integ $y = -2\frac{p^{-1}}{-1} + c$

$$y = \frac{2}{p} + c$$

$$y - c = \frac{2}{p}$$

$$\frac{1}{p} = \frac{y-c}{2}$$

Substituting the value of p in (1)

$$x = \left(\frac{y-c}{2}\right)^2 + 1$$

$$x - 1 = \frac{1}{4}(y-c)^2$$

$4(x-1) = (y-c)^2$ is the reqd. sol^n

Prob [3] Solve $p^2 - 2xp + 1 = 0$

Sol^n $2xp = p^2 + 1$

$$x = \frac{1}{2p}(p^2+1)$$

$$x = \frac{p}{2} + \frac{1}{2p} \quad \text{— (1)}$$

$$= f(p)$$

$\Rightarrow$ solvable for x

$$x = \frac{p}{2} + \frac{1}{2p}$$

diff. w.r.to y bothsides

$$\frac{dx}{dy} = \frac{1}{2}\frac{dp}{dy} - \frac{1}{2p^2}\frac{dp}{dy}$$

$$\frac{1}{p} = \left(\frac{1}{2} - \frac{1}{2p^2}\right)\frac{dp}{dy}$$

$$\frac{1}{p} = \frac{1}{2}\left(1-\frac{1}{p^2}\right)\frac{dp}{dy}$$

$$dy = \frac{1}{2}p\left(1-\frac{1}{p^2}\right)dp$$

$$= \frac{1}{2}\left(p-\frac{1}{p}\right)dp$$

Integ, $y = \frac{1}{2}\left(p^2/2 - \log p\right) + c$ — (2)

The p-eliminant of (1) and (2) constitutes the solution.

Prob [4] Solve $4yp^2 - 2xp + y = 0$

Given $4yp^2 + y = 2xp$

$$2x = 4yp + \frac{y}{p} \quad -(1)$$

$$= f(y, p)$$

$\Rightarrow$ Solvable for x

$\because$ $2x = 4yp + y/p$

Diff. w.r.to y both sides

$$2\frac{dx}{dy} = 4p + 4y\frac{dp}{dy} + \frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy}$$

$$2/p = 4p + 4y\frac{dp}{dy} + \frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy}$$

$$\frac{1}{p} - 4p = y\left(4 - \frac{1}{p^2}\right)\frac{dp}{dy}$$

$$\frac{1-4p^2}{p} = y\left(\frac{4p^2-1}{p^2}\right)\frac{dp}{dy}$$

$$\frac{1-4p^2}{p} + \frac{y}{p}\left(\frac{1-4p^2}{p}\right)\frac{dp}{dy} = 0$$

$$\frac{y}{p}\left(\frac{1-4p^2}{p}\right)dp$$

$$\left(\frac{1-4p^2}{p}\right)\left(1 + \frac{y}{p}\frac{dp}{dy}\right) = 0$$

When $1 + \frac{y}{p}\frac{dp}{dy} = 0$

$$\frac{dy}{y} + \frac{dp}{p} = 0$$

Integ, $\log y + \log p = \log c$

$$py = c$$

$$p = c/y$$

Putting $p = c/y$ in (1)

$$2x = 4y \times \frac{c}{y} + \frac{y}{c/y}$$

$$2x = 4c + y^2/c$$

$2cx = 4c^2 + y^2$ is the reqd. Sol^n.

$\frac{1-4p^2}{p}$ is beyond the scope of this problem.

Prob [5] $p^3 - p(y+3) + x = 0$

Sol^n $p^3 - p(y+3) = -x$ — (1)

$\Rightarrow$ $x = p(y+3) - p^3 = f(y, p)$

$\Rightarrow$ Solvable for x

$\because$ $x = p(y+3) - p^3$

Diff. w.r. to y b.s

$$\frac{dx}{dy} = \frac{dp}{dy}(y+3) + p - 3p^2\frac{dp}{dy}$$

$$\frac{1}{p} = (y+3-3p^2)\frac{dp}{dy} + p$$

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$$\frac{1}{p} - p = \frac{dp}{dy}\left(y+3-3p^2\right)$$

$$\frac{1-p^2}{p} = \left(y+3-3p^2\right)\cdot\frac{dp}{dy}$$

$$\frac{(1-p^2)}{p}\,\cancel{dp}\,\frac{dy}{dp} = y+3-3p^2$$

$$\left(\frac{1-p^2}{p}\right)\frac{dy}{dp} - y = 3(1-p^2)$$

$$\frac{dy}{dp} - \frac{p}{1-p^2}\,y = 3p$$

Which is clearly a linear differential eqn in y

$$I.F = e^{\int P\,dp} = e^{\int \frac{-p}{1-p^2}dp}$$

$$= e^{\frac{1}{2}\int \frac{-2p}{1-p^2}dp}$$

$$= e^{\frac{1}{2}\log(1-p^2)}$$

$$= e^{\log\sqrt{1-p^2}} = \sqrt{1-p^2}$$

Its Solution is given by

$$y\cdot I.F = \int Q\,(IF)\,dp + C$$

$$y.\sqrt{1-p^2} = \int 3p\sqrt{1-p^2}\,dp + C$$

$$= -\frac{3}{2}\int z^2\,dz \qquad \text{Putting } 1-p^2 = z, \; -2p\,dp = dz$$

$$= -\frac{3}{2}\times\frac{z^3}{3} + C$$

$$y\sqrt{1-p^2} = -\frac{1}{2}(1-p^2)^{3/2} + C \quad \text{--- (2)}$$

The p-eliminant of (1) & (2) will give the reqd soln

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Prob [6] Solve $y = yp^2 + 2px$

Soln Given $y = yp^2 + 2px$

W.O (2)

$$2px = y - yp^2$$

$$x = \frac{1}{2p}y - \frac{yp^2}{2p}$$

$$= f(p, y)$$

$\Rightarrow$ solvable for x

$$\because x = \frac{y}{2p} - \frac{1}{2}py$$

$$x = \left(\frac{1}{2p} - \frac{p}{2}\right)y \quad \text{--- (1)}$$

Diff. w.r.to y, both sides, we get

$$\frac{dx}{dy} = \left(\frac{1}{2p} - \frac{p}{2}\right)\cdot 1 + y\left(-\frac{1}{2p^2} - \frac{1}{2}\right)\frac{dp}{dy}$$

$$\frac{1}{p} = \left(\frac{1}{2p} - \frac{p}{2}\right) = -\left(\frac{1}{2p^2} + \frac{1}{2}\right)y\frac{dp}{dy}$$

$$\left(\frac{1}{2p^2} + \frac{1}{2}\right)y\cdot\frac{dp}{dy} = \frac{1}{2p} - \frac{p}{2} - \frac{1}{p}$$

$$= -\frac{1}{2p} - \frac{p}{2}$$

$$= -p\left(\frac{1}{2} + \frac{1}{2p^2}\right)$$

$$\left(\frac{1}{2} + \frac{1}{2p^2}\right)\left(\frac{dp}{dy}\cdot(y) + p\right) = 0$$

When $y\frac{dp}{dy} + p = 0$ $\qquad \frac{1}{2}\left(\frac{1+p^2}{p^2}\right) \neq 0$

$$y\frac{dp}{dy} = -p$$

$$y\,dp + p\,dy = 0$$

$$d(yp) = 0$$

$$py = c \Rightarrow p = c/y \quad \text{--- (2)}$$

sub. in (2), $x = \left(\frac{1}{2c/y} - \frac{c}{2y}\right)y$

$$x = \left(\frac{y}{2c} - c/2y\right)y = \frac{y^2 - c^2}{2cy}\cdot y$$

$$2cx = y^2 - c^2$$

$y^2 = 2cx + c^2$ is the reqd. soln

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Prob [7] Solve $y - 2px + ayp^2 = 0$

Sol^n Given $y - 2px + ayp^2$

$$2px = y + ayp^2$$

$$x = \frac{y}{2p} + \frac{a}{2}py = f(p, y)$$

$\Rightarrow$ Solvable for x

$$\because\ x = \frac{1}{2}\frac{y}{p} - \frac{a}{2}py \quad -(1)$$

Diff. w.r.to y b.s

$$\frac{dx}{dy} = \frac{1}{2}\cdot\frac{1}{p} - \frac{y}{2p^2}\frac{dp}{dy} - \frac{a}{2}p - \frac{ay}{2}\frac{dp}{dy}$$

$$\Rightarrow \frac{1}{p} = \frac{1}{2p} - \frac{1}{2}ap - \frac{y}{2p^2}\frac{dp}{dy} - \frac{a}{2}y\frac{dp}{dy}$$

$$\Rightarrow \frac{1}{p} - \frac{1}{2p} + \frac{1}{2}ap = -\frac{1}{2}\left(\frac{y}{p^2} + ay\right)\frac{dp}{dy}$$

$$\Rightarrow \frac{1}{2p} + \frac{1}{2}ap = -\frac{1}{2}\left(\frac{y}{p^2} + ay\right)\frac{dp}{dy}$$

$$\Rightarrow \frac{1}{\cancel{2}}\left(\frac{1+ap^2}{\cancel{p}}\right) = -\frac{y}{\cancel{2}}\left(\frac{1+ap^2}{p^{\cancel{2}}}\right)\frac{dp}{dy}$$

$$\Rightarrow (1+ap^2)\left(1 + \frac{y}{p}\frac{dp}{dy}\right) = 0$$

When $1 + \frac{y}{p}\frac{dp}{dy} = 0$ | But $1 + ap^2 \neq 0$

$$pdy + ydp = 0$$

$$d(py) = 0$$

Integ, $py = c$

$$p = y/c$$

from (1) $x = \frac{\cancel{y}}{2}\times\frac{c}{\cancel{y}} - \frac{a}{2}y\cdot\frac{y}{c}$

$$2cx = c^2 - ay^2$$

$\Rightarrow 2cx = c^2 - ay^2$ is reqd. solution.

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Prob [8] $ayp^2 + (2x-b)p - y = 0$

Sol^n. $ayp^2 + (2x-b)p - y = 0$

$$\Rightarrow (2x-b)p = y + ayp^2$$

$$\Rightarrow 2xp = y + bp + ayp^2$$

$$\therefore\ 2x = \frac{y}{p} + b + ayp \quad -(1)$$

$$= f(b, p)$$

$\Rightarrow$ Solvable for x

Diff (1) w.r.to y both sides

$$\Rightarrow \frac{2dx}{dy} = \frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy} + 0 + a\left[p + y\frac{dp}{dy}\right]$$

$$\Rightarrow \frac{2}{p} - \frac{1}{p} = -\frac{y}{p^2}\frac{dp}{dy} + ap + ay\frac{dp}{dy}$$

$$\Rightarrow \frac{1}{p} + ap = -\left(a + \frac{1}{p^2}\right)y\frac{dp}{dy}$$

$$\Rightarrow \frac{1+ap^2}{\cancel{p}} = -\frac{(ap^2+1)}{p^{\cancel{2}}}\cdot y\frac{dp}{dy}$$

$$\Rightarrow (1+ap^2)\left(1 + \frac{y}{p}\frac{dp}{dy}\right) = 0$$

Here $1 + \frac{y}{p}\frac{dp}{dy} = 0$

$$\Rightarrow pdy + ydp = 0$$

$$\therefore\ d(py) = 0$$

Integ, $py = c \Rightarrow p = c/y$

Sub. $p = c/y$ in (1)

$$2x = y\cdot\frac{y}{c} + b + a\cancel{y}\cdot\frac{c}{\cancel{y}}$$

$$= \frac{y^2}{c} + b + ac$$

$$2cx = y^2 + bc + ac^2$$

$ac^2 + (2x-b)c - y^2 = 0$ is the reqd. Solution

Prob [9] Solve $y = 2px + y^2p^3$

Given $y = 2px + y^2p^3$

$$2px = y - y^2p^3$$

$$2x = \frac{y}{p} - y^2p^2 = f(p,y)$$

⇒ solvable for x

$$\because 2x = \frac{y}{p} - y^2p^2 \quad -(1)$$

Diff. w.r.to y, we get

$$2\frac{dx}{dy} = \frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy} - 2yp^2 - y^2\,2p\frac{dp}{dy}$$

$$\frac{2}{p} - \frac{1}{p} + 2yp^2 = \left(-\frac{y}{p^2} - 2y^2p\right)\frac{dp}{dy}$$

$$\frac{1}{p} + 2yp^2 = -y\left(\frac{1}{p^2} + 2y^2p\right)\frac{dp}{dy}$$

$$\frac{1+2yp^3}{\cancel{p}} = -\frac{y}{p^{\cancel{2}}}\left(1+2y^2p^3\right)\frac{dp}{dy}$$

$$(1+2yp^3)\left(1+\frac{y}{p}\frac{dp}{dy}\right) = 0$$

$$1+\frac{y}{p}\frac{dp}{dy} = 0$$

$$\Rightarrow p\,dy + y\,dp = 0$$

$$d(py) = 0$$

$$py = c \Rightarrow p = c/y$$

Putting $p = c/y$ in the given eqn

$$y = 2x\times\frac{c}{y} + y^2\times\frac{c^3}{y^3}$$

$$y^2 = 2cx + c^3$$

$$\therefore y^2 = 2cx + c^3$$

Which is the reqd. Solution

[10] Solve $p^3 - 4xyp + 8y^2 = 0$

Given $p^3 - 4xyp + 8y^2 = 0$

$$\Rightarrow 4xyp = p^3 + 8y^2$$

$$4x = \frac{p^2}{y} + \frac{8y}{p}$$

$$x = \frac{p^2}{4y} + \frac{2y}{p} \quad -(1)$$

$$= f(y,p)$$

⇒ solvable for x

Diff w.r.to y,

$$\frac{dx}{dy} = \frac{1}{4y}\left[2p\frac{dp}{dy}\right] - \frac{p^2}{4}\frac{1}{y^2} + 2\left[\frac{1}{p} - \frac{y}{p^2}\frac{dp}{dy}\right]$$

$$-\frac{2}{p} + \frac{1}{p} + \frac{p^2}{4y^2} = \frac{p}{2y}\frac{dp}{dy} - \frac{2y}{p^2}\frac{dp}{dy}$$

$$-\frac{1}{p} + \frac{p^2}{4y^2} = \left(\frac{p}{2y} - \frac{2y}{p^2}\right)\frac{dp}{dy}$$

$$\frac{p^3 - 4y^2}{\cancel{p}\cdot 4y^{\cancel{2}}} = \frac{p^3 - 4y^2}{\cancel{2y}\cdot p^{\cancel{2}}}\frac{dp}{dy}$$

$$(p^3 - 4y^2)\left(\frac{1}{2y} - \frac{1}{p}\frac{dp}{dy}\right) = 0$$

But $p^3 - 4y^2 \neq 0$

So
$$\frac{1}{2y} - \frac{1}{p}\frac{dp}{dy} = 0$$

$$\frac{1}{2y} = \frac{1}{p}\frac{dp}{dy}$$

$$\frac{dy}{y} = \frac{2\,dp}{p}$$

Integ., $2\log p = \log y + \log c$

$$\log p^2 = \log cy$$

$$p^2 = cy$$

Putting $p^2 = cy$ in (1)

$$x = \frac{cy}{4y} + \frac{2y}{\sqrt{cy}}$$

$$x - \frac{cy}{4y} = \frac{2y}{\sqrt{c}\sqrt{y}}$$

$$\frac{4xy - cy}{4y} = \frac{2y}{\sqrt{cy}}$$

$$\frac{(4xy - cy)^2}{y^2} = \frac{64y^2}{cy}$$

$$\boxed{c(4x-c)^2 = 64y}$$

Reqd. soln