SOLVABLE FOR y

— PROBLEMS —

Ex – 6(B) (L.P)

(1) Solve: $y = p\cos p - \sin p$

Sol^n Given $y = p\cos p - \sin p$ — ①

Diff. w.r.to x b.s, we get

$$\frac{dy}{dx} = \cancel{\cos p.\frac{dp}{dx}} - p\sin p\frac{dp}{dx} - \cancel{\cos p\frac{dp}{dx}}$$

$$p = -p\sin p\frac{dp}{dx}$$

$$\Rightarrow p + p\sin p\frac{dp}{dx} = 0$$

$$\Rightarrow p\left(1+\sin p\frac{dp}{dx}\right) = 0$$

$$\Rightarrow 1+\sin p\frac{dp}{dx} = 0, \quad \because p \neq 0$$

$$\Rightarrow dx + \sin p\,dp = 0$$

Integ., $\int dx + \int \sin p\,dp = c$

$$\Rightarrow x - \cos p = c$$

$$\therefore x = \cos p + c \quad — ②$$

The p-eliminant of ① & ② will give the solution.

② Solve: $y = 2p + 3p^2$

Sol^n Given $y = 2p + 3p^2$ —(1)

Diff w.r.to x both sides, we get

$$\frac{dy}{dx} = 2\frac{dp}{dx} + 6p\frac{dp}{dx}$$

$$p = (2+6p)\frac{dp}{dx}$$

$$dx = \left(\frac{2}{p} + 6\right)dp$$

Integrating,

$$x = 2\log p + 6p + c \quad — ②$$

The p-eliminant of ① & ② will constitute the solution.

(3) Solve: $y = 2px + 4xp^2$

Sol^n Given: $y = 2px + 4xp^2$ —(1)

Diff. w.r.to x both sides, we get

$$\frac{dy}{dx} = 2p + 2x\frac{dp}{dx} + 4p^2 + 4x.2p\frac{dp}{dx}$$

$$\Rightarrow p = 2p + 2x(1+4p)\frac{dp}{dx} + 4p^2$$

$$\Rightarrow 0 = p + 4p^2 + 2x(1+4p)\frac{dp}{dx}$$

$$\Rightarrow 0 = p(1+4p) + 2x(1+4p)\frac{dp}{dx}$$

$$0 \Rightarrow (1+4p)\left(p + \frac{dp}{dx}(2x)\right)$$

$$\Rightarrow (1+4p)\left(p + 2x\frac{dp}{dx}\right) = 0$$

when $1+4p = 0 \Rightarrow 1 + 4\frac{dy}{dx} = 0$

$$dx + 4dy = 0$$

$$x + 4y = c \quad — ②$$

When $p + 2x\frac{dp}{dx} = 0$

$$\Rightarrow 2x\,dp + p\,dx = 0$$

$$\Rightarrow 2\frac{dp}{p} + \frac{dx}{x} = 0$$

Integ $2\log p + \log x = \log c$

$$\log p^2 + \log x = \log c$$

$$\log xp^2 = \log c$$

$$\Rightarrow xp^2 = c$$

$$\therefore p = \sqrt{\frac{c}{x}}$$

putting $p = \sqrt{c/x}$ in (1),

$$\Rightarrow y = 2x\sqrt{\frac{c}{x}} + 4x \cdot \frac{c}{\cancel{x}}$$

$$\Rightarrow y - 4c = 2\sqrt{c \cdot x}$$

$$\therefore (y-4c)^2 = 4cx$$

Which is the required soln

[4] Solve $y = x(p+p^3)$

Soln - Given $y = x(p+p^3)$ — (1)

Diff. w.r.to x both sides,

$$\cancel{\frac{dy}{dx}} = \cancel{p + x\frac{dp}{dx}}$$

$$\Rightarrow \frac{dy}{dx} = (p+p^3) + x\left(\frac{dp}{dx} + 3p^2\frac{dp}{dx}\right)$$

$$\Rightarrow p = p + p^3 + x(1+3p^2)\frac{dp}{dx}$$

$$\Rightarrow -p^3 = x(1+3p^2)\frac{dp}{dx}$$

$$\Rightarrow -\frac{dx}{x} = \frac{1+3p^2}{p^3}\,dp$$

$$\Rightarrow \frac{dx}{x} + \left(\frac{1}{p^3} + \frac{3p^2}{p^3}\right)dp = 0$$

Integ, $\log x + \left(-\frac{1}{2p^2}\right) + 3\log p \mp \log c = 0$

$$\log x + \log p^3 + \log c = \frac{1}{2p^2}$$

$$\log(cxp^3) = \frac{1}{2p^2}$$

$$cxp^3 = e^{+1/2p^2} \quad \text{— (2)}$$

The p-eliminant of (1) and (2) will constitute the solution of the given equation.

[5]. Solve $p^2 + p = y/x$

Given $p^2 + p = y/x$

$$\Rightarrow y = x(p+p^2)$$

Diff. w.r.to x bothsides

$$\frac{dy}{dx} = (p+p^2) + x\left(\frac{dp}{dx} + 2p\frac{dp}{dx}\right)$$

$$\Rightarrow \cancel{p} = \cancel{p} + p^2 + x(1+2p)\frac{dp}{dx}$$

$$\Rightarrow -p^2 = x(1+2p)\frac{dp}{dx}$$

$$\Rightarrow -\frac{dx}{x} = \frac{1+2p}{p^2}\,dp$$

$$\Rightarrow \frac{dx}{x} + \left(\frac{1}{p^2} + \frac{2}{p}\right)dp = 0$$

Integ, $\log x + \left(-\frac{1}{p} + 2\log p\right) = \log c$

$$\log x - \frac{1}{p} + \log p^2 = \log c$$

$$\frac{1}{p} = \log p^2 x - \log c$$

$$= \log\left(\frac{p^2 x}{c}\right)$$

$$x p^2/c = e^{1/p}$$

$$x = c\,p^{-2} e^{1/p} \quad -(2)$$

The p-eliminant of (1) & (2) constitute the solution of the equation.

[6] Solve: $y = x(p^2 - 2p + 2)$

Soln — Given: $y = x(p^2 - 2p + 2)$

Diff. w.r.to x both sides

$$\frac{dy}{dx} = (p^2 - 2p + 2) + x(2p-2)\frac{dp}{dx}$$

$$\Rightarrow p = p^2 - 2p + 2 + 2x(p-1)\frac{dp}{dx}$$

$$-(p^2 - 3p + 2) = 2x(p-1)\frac{dp}{dx}$$

$$\Rightarrow -(p-1)(p-2) = 2x(p-1)\frac{dp}{dx}$$

$$\Rightarrow (p-1)\left\{(p-2) + 2x\frac{dp}{dx}\right\} = 0$$

$\Rightarrow$ When $p-1=0 \Rightarrow p=1$ and substituting in (1), we get $y = x(1-2+2)$

$\therefore y = x$ is the reqd. soln

When $(p-2) + 2x\frac{dp}{dx} = 0$

$$\Rightarrow \frac{p-2}{2} + x\frac{dp}{dx} = 0$$

$$\Rightarrow \frac{1}{2}(p-2) = -x\frac{dp}{dx}$$

$$\Rightarrow \frac{1}{2}\left(\frac{dx}{x}\right) = -\frac{dp}{p-2}$$

$$\Rightarrow \frac{1}{2}\frac{dx}{x} + \frac{dp}{p-2} = 0$$

Integ, $\frac{1}{2}\log x + \log(p-2) = \log K$

$$\Rightarrow \log\sqrt{x}(p-2) = \log K$$

$$\Rightarrow \sqrt{x}(p-2) = K$$

$$\Rightarrow p - 2 = K/\sqrt{x}$$

$$\therefore p = 2 + K/\sqrt{x} \quad -(2)$$

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Putting $p = 2 + K/\sqrt{x}$ in (1), we get

$$y = x\left\{\left(2 + K/\sqrt{x}\right)^2 - 2\left(2 + K/\sqrt{x}\right) + 2\right\}$$

$$= x\left\{\cancel{4} + \frac{4K}{\sqrt{x}} + \frac{K^2}{x} - \cancel{4} - \frac{2K}{\sqrt{x}} + 2\right\}$$

$$= x\left(\frac{2K}{\sqrt{x}} + \frac{K^2}{x} + 2\right)$$

$$\Rightarrow y = 2K\sqrt{x} + K^2 + 2x$$

$$\Rightarrow y - 2x - K^2 = 2K\sqrt{x}$$

$$\therefore (y - 2K - K^2)^2 = 4K^2 x \quad \text{(on squaring)}$$

which is the reqd. solution

[7] $xp^2 - 2yp - x = 0$

Soln

Given $xp^2 - 2yp - x = 0$

$$2yp = xp^2 - x$$

$$y = \frac{(p^2 - 1)x}{2p}$$

$$y = \left(\frac{p}{2} - \frac{1}{2p}\right)x \quad -(1)$$

Diff. w.r.to x b.s

$$\frac{dy}{dx} = p = \left(\frac{p}{2} - \frac{1}{2p}\right) + x\left(\frac{1}{2} + \frac{1}{2p^2}\right)\frac{dp}{dx}$$

~~$= p(\frac{1}{2}$~~

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$$\Rightarrow p - \frac{p}{2} + \frac{1}{2p} = x\left(\frac{1}{2} + \frac{1}{2p^2}\right)\frac{dp}{dx}$$

$$\frac{p}{2} + \frac{1}{2p} = x\left(\frac{1}{2} + \frac{1}{2p^2}\right)\frac{dp}{dx}$$

$$p\left(\frac{1}{2} + \frac{1}{2p^2}\right) = x\left(\frac{1}{2} + \frac{1}{2p^2}\right)\frac{dp}{dx}$$

$$\left(\frac{1}{2} + \frac{1}{2p^2}\right)\left(p - x\frac{dp}{dx}\right) = 0$$

when

$$\frac{1}{2} = -\frac{1}{2}p^2$$

$$p^2 = -1$$

$$p = \sqrt{-1} = i$$

rejected

when $p - x\dfrac{dp}{dx} = 0$

$$p = x\frac{dp}{dx}$$

$$\frac{dx}{x} = \frac{dp}{p}$$

Integ. $\log x + \log c = \log p$

$$\log cx = \log p$$

$$p = cx$$

Putting $p = cx$ in (1),

$$y = \left(\frac{cx}{2} - \frac{1}{2cx}\right)x = \left(\frac{c^2x^2 - 1}{2cx}\right)x$$

$$\therefore 2cy = c^2x^2 - 1$$

which is the reqd. solution.

[8] Solve $4xp^2 - 8yp - x = 0$

Sol^n: Given $4xp^2 - 8yp - x = 0$

$$8yp = 4xp^2 - x$$

$$y = \frac{1}{2}xp - \frac{1}{8p}x \qquad -(1)$$

Diff. w.r.to x both sides

$$\frac{dy}{dx} = \frac{1}{2}p + \frac{1}{2}x\frac{dp}{dx} - \frac{1}{8p} + \frac{1 \cdot x}{8p^2}\frac{dp}{dx}$$

$$\Rightarrow p = \frac{1}{2}p - \frac{1}{8p} + \frac{1}{2}x\frac{dp}{dx}\left(1+\frac{1}{4p^2}\right)$$

$$\Rightarrow p - \frac{1}{2}p + \frac{1}{8p} = \frac{1}{2}x\left(1+\frac{1}{4p^2}\right)\frac{dp}{dx}$$

$$\Rightarrow \frac{p}{2} + \frac{1}{8p} = \frac{1}{2}x\left(1+\frac{1}{4p^2}\right)\frac{dp}{dx}$$

$$\Rightarrow \left(\frac{4p^2+1}{8p}\right) = \frac{1}{2}x\left(1+\frac{1}{4p^2}\right)\frac{dp}{dx}$$

$$\Rightarrow \frac{4p^2+1}{8p} = \frac{1}{2}x\left(\frac{4p^2+1}{4p^2}\right)\frac{dp}{dx}$$

$$\Rightarrow \frac{(4p^2+1)}{8p}\left[1 - \frac{x}{p}\frac{dp}{dx}\right] = 0$$

But when $4p^2+1 = 0$

$$p^2 = -\tfrac{1}{4}$$

$$\Rightarrow p = \sqrt{-\tfrac{1}{4}} = \frac{i}{2}$$

But p is real, hence rejected

& $1 - \frac{x}{p}\frac{dp}{dx} = 0$

$$1 = \frac{x}{p}\frac{dp}{dx}$$

$$\frac{dx}{x} = \frac{dp}{p}$$

Int. $\log p = \log x + \log c$

$$p = cx \qquad -(2)$$

Eliminating p between (1) & (2) we get

$$y = \frac{1}{2}x \cdot cx - \frac{1}{8cx}x = \frac{1}{2}cx^2 - \frac{1}{8c}$$

$\therefore y = \frac{c}{2}x^2 - \frac{1}{8c}$ is the reqd. solution.

Notes

(9) $y = p^2x + p^3$

Sol^n Given $y = p^2x + p^3 \quad -(1)$ Type $y = f(x,p)$ $\Rightarrow$ solvable for y

diff w.r.to x

$$\frac{dy}{dx} = 2p\frac{dp}{dx}\cdot x + p^2 + 3p^2\frac{dp}{dx}$$

$$= p^2 + (2px + 3p^2)\frac{dp}{dx}$$

$$p = p^2 + (2px + 3p^2)\frac{dp}{dx}$$

$$1 = p + (2x+3p)\frac{dp}{dx} \Rightarrow 1 - p = (2x+3p)\frac{dp}{dx}$$

$$\frac{dx}{dp} = \frac{2x+3p}{1-p}$$

$$= \frac{2x}{1-p} + \frac{3p}{1-p}$$

$$\frac{dx}{dp} - \frac{2x}{1-p} = \frac{3p}{1-p}$$

$$\frac{dx}{dp} + \frac{2x}{p-1} = -\frac{3p}{p-1}$$

$\left[\begin{array}{l} P = 2/(p-1) \\ Q = -3p/(p-1) \end{array}\right.$

which is clearly a linear diff. eq^n in x

$$\text{I.F} = e^{\int P\,dp} = e^{\int \frac{2}{p-1}dp}$$

$$= e^{2\log(p-1)} = e^{\log(p-1)^2}$$

$$= (p-1)^2$$

$\frac{dx}{dy} + Px = Q$ → linear

$\text{IF} = e^{\int P\,dy}$

Sol^n: $x(\text{IF}) = \int Q(\text{I.F})\,dx + c$

Its solution is

$$x\cdot(\text{I.F}) = \int -\frac{3p}{p-1} \times (p-1)^2\,dp + c$$

$$x\cdot(p-1)^2 = -3\int p(p-1)\,dp + c$$

$$= -3\frac{p^3}{3} + \frac{3}{2}p^2 + c$$

$$x(p-1)^2 = -p^3 + \frac{3}{2}p^2 + c \qquad -(2)$$

The p-eliminant of (1) & (2) will give the solution of the given d.e. Ans

Notes Date

[10] Solve $y = (1+p)x + p^2 = f(x,p)$

$\Rightarrow$ Solvable for y

Given $y = (1+p)x + p^2$ — ①

Diff. w.r.to x. both sides

$$\frac{dy}{dx} = (1+p) + x\left(\frac{dp}{dx}\right) + 2p\frac{dp}{dx}$$

$$\cancel{p} = 1 + \cancel{p} + (x+2p)\frac{dp}{dx}$$

$$-1 = (x+2p)\frac{dp}{dx}$$

$$-\frac{dx}{dp} = x + 2p$$

$$-\frac{dx}{dp} - x = 2p$$

$$\frac{dx}{dp} + x = -2p$$

Which is clearly a Linear diff eq^n in x.

$$I.F = e^{\int P dp} = e^{\int 1 dp} = e^p$$

Its Solution

$$x.(I.F) = \int -2p\,(I.F)\,dp + c$$

$$x\,e^p = -\int 2p\,e^p\,dp + c$$

$$= -2\left[p.e^p - \int 1.e^p dp\right] + c$$

$$= -2[p\,e^p - e^p] + c$$

$$= -2pe^p + e^p + c$$

$$x = -2p + 1 + c\,e^{-p} \quad \text{— ②}$$

The p-eliminant of ① and ② will give the required sol^n.

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W.O (4) Solve $y = (1+p)x + ap^2$

$= f(x,p)$

$\Rightarrow$ Solvable for y

$\because y = (1+p)x + ap^2$ — ①

Diff. w.r.to x both sides,

$$\frac{dy}{dx} = p = (1+p) + x\left(\frac{dp}{dx}\right) + a.2p\frac{dp}{dx}$$

$$-1 = (x+2pa)\frac{dp}{dx}$$

$$-\frac{dx}{dp} = x + 2ap$$

$$\Rightarrow -\frac{dx}{dp} - x = 2ap$$

$$\Rightarrow \frac{dx}{dp} + x = -2ap$$

Which is clearly a L.D.E in x

$$I.F = e^{\int P dp} = e^{\int 1 dp} = e^p \quad [\because P = 1$$

Its Solution

$$x.(I.F) = \int Q.(I.F)\,dp + c$$

$$x.\,e^p = \int -2ap\,e^p\,dp + c$$

$$= -2a(p-1)e^p + c$$

$$x = -2a(p-1) + c.e^{-p} \quad \text{— ②}$$

The p eliminant of ① & ② will give the required solution.