introduced a complex ...

a and b are real numbers.

Complex Number:- A complex number $x+iy$ may be defined as an orderd pair of real numbers (x, y) denoted by z i.e, $z = (x, y)$, x is called the real part of z and y is called the imaginary part of z. The real part of z is denoted by $Re(z)$ and the imaginary part by $Im(z)$. i.e, $x = Re(z)$, $y = Im(z)$.

Purely real and purely imaginary ~~part~~ complex numbers — A complex number z is purely real if its imaginary part is zero i.e, $Im(z) = 0$ and purely imaginary if its real part is zero i.e., $Re(z) = 0$

Set of Complex numbers: The set of all Complex numbers is denoted by C i.e.,

$$C = \{z = a + ib = (a, b) : a \in R, b \in R\}$$

Notes

Since a real number 'a' can be written as $a + i.0$. Therefore, every real number is a Complex number. Hence $R \subset C$

Equality of Complex numbers: Two Complex numbers are equal if real part of one is equal to the real part of the other, and the imaginary part of one is equal to the imaginary part of the other.

5 WED

Thus two complex numbers $z_1 = (x_1, y_1)$ and $z_2 = (x_2, y_2)$ are equal iff $x_1 = x_2$ and $y_1 = y_2$

Conjugate of a Complex number: The Conjugate of a complex number $z = x+iy$ is $x-iy$ and is usually denoted by $\bar{z}$. Thus if $z = (x, y)$, then $\bar{z} = (x, -y)$

FUNDAMENTAL OPERATIONS WITH COMPLEX NUMBERS

6 THU

(i) Addition : $z_1 + z_2 = (x_1+x_2,\ y_1+y_2)$

(ii) Subtraction : $z_1 - z_2 = (x_1-x_2,\ y_1-y_2)$

(iii) Multiplication: $z_1 \cdot z_2 = (x_1x_2 - y_1y_2,\ x_1y_2 + x_2y_1)$

(iv) Division : $\frac{z_1}{z_2} = \frac{(x_1, y_1)}{(x_2, y_2)} = \left(\frac{x_1x_2+y_1y_2}{x_2^2+y_2^2},\ \frac{y_1x_2 - x_1y_2}{x_2^2+y_2^2}\right)$

Provided $x_2^2+y_2^2 \neq 0$

FUNDAMENTAL LAWS OF ADDITION and MULTIPLICATION

COMMUTATIVE LAW FOR ADDITION –

$$\begin{aligned} z_1 + z_2 &= (x_1, y_1) + (x_2, y_2) = (x_1+x_2,\ y_1+y_2) \\ &= (x_2+x_1,\ y_2+y_1) \\ &= (x_2, y_2) + (x_1, y_1) \\ &= z_2 + z_1 \end{aligned}$$

Notes $\therefore z_1 + z_2 = z_2 + z_1, \quad \forall\ z_1, z_2 \in C$

MODULUS OF A COMPLEX NUMBER

13 THU The modulus of a complex number $z = a+ib$ is denoted by $|z|$ and is defined as

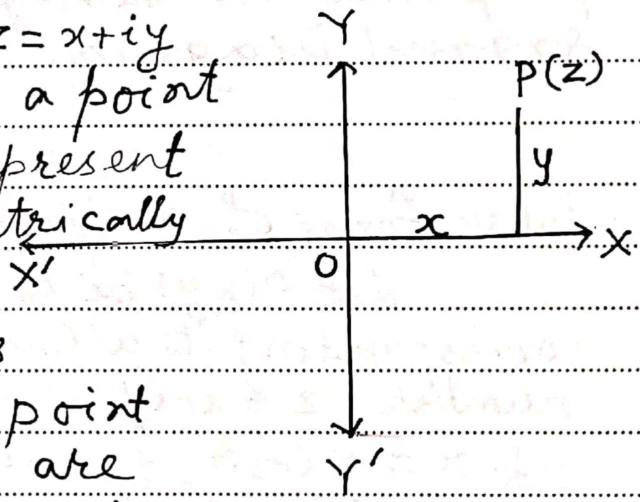
$$|z| = +\sqrt{a^2+b^2} = +\sqrt{(Re(z))^2+(Im(z))^2}$$

Clearly, $|z| \geq 0$, $\forall z \in C$.

If $|z| = 0 \Leftrightarrow x=0,\ y=0$

Geometrically, $|z|$ is the distance of the point z ~~written~~ from the origin.

GEOMETRICAL REPRESENTATION OF A COMPLEX NUMBER



A complex number $z = x+iy$

14 FRI can be represented by a point (x,y) on the plane. To represent a complex number, geometrically we take two mutually perpendicular straight lines $X'OX$ and $Y'OY$. Now plot a point whose x and y co-ordinates are respectively the real and imaginary parts of z. This point $P(x,y)$ represents the complex number $z = x+iy$

If a complex number is purely real, then its imaginary part is zero. Therefore, a purely real number is represented by a point on x-axis

Notes

A purely imaginary number is represented by a point on y-axis. That is why x-axis is known as the real axis and y-axis as the imaginary axis.

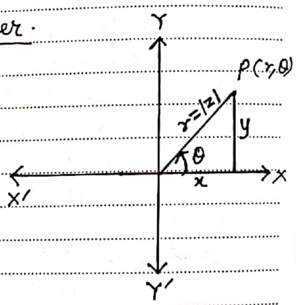
Conversely, if P(x,y) is be a point in the plane, then the point P(x,y) represents a complex number $z = x+iy$

Thus, for every complex number $z = x+iy$ there exists uniquely a point (x,y) on the plane and for every point (x,y) of the plane there exists uniquely a complex number $z = x+iy$.

The plane in which we represent a complex number geometrically is known as the Complex plane or Argand plane or the Gaussian Plane. The point P plotted on the Argand plane, is called the Argand Diagram.

16 SUN

Polar forms of a Complex Number.



Let P(x,y) be a point corresponding to a complex number $z = x+iy$

If $x = r\cos\theta$, $y = r\sin\theta$ then $r = +\sqrt{x^2+y^2}$ is called the modulus of a complex number and $\theta = \tan^{-1}(y/x)$ is called the argument of a complex number.

$$z = x+iy = r(\cos\theta + i\sin\theta) = re^{i\theta}.$$

Notes It is called the polar form of a complex number z or Eulerian form of z

r is also called the absolute value of z and θ is also called the amplitude of z

i.e., $r = |z|$, $\theta = amp(z)$ or $arg(z)$.

Note – The angle θ has infinitely many values differing by multiples of 2π. The value of θ s.t $-\pi \le \theta \le \pi$ is called principal value of the amplitude or principal argument.

Properties of MODULUS –

If z_1, z_2, z_3 are three complex numbers, then

(i) $|z| = 0 \Leftrightarrow z = 0 \Leftrightarrow Re(z) = Im(z) = 0$

(ii) $|z| = |\bar{z}| = |-z|$

(iii) $-|z| \le Re(z) \le |z|$, $-|z| \le Im(z) \le |z|$

(iv) $z\bar{z} = |z|^2$

18 TUE

(v) $|z_1 z_2| = |z_1|\,|z_2|$

(vi) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $z_2 \ne 0$

(vii) $|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z}_2)$

(viii) $|z_1-z_2|^2 = |z_1|^2 + |z_2|^2 - 2Re(z_1\bar{z}_2)$

(ix) $|z_1+z_2|^2 + |z_1-z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

(x) $|z^n| = |z|^n$

Proof: Let $z = (x, y)$

(i) $\because |z| = 0$

$\Leftrightarrow \sqrt{x^2+y^2} = 0$

$\Leftrightarrow x^2+y^2 = 0$

$\Leftrightarrow x = 0$ and $y = 0 \Leftrightarrow z = 0$

$\Leftrightarrow Re(z) = 0 = Im(z)$

Notes

(ii) Let $z = (x, y)$ then $\bar{z} = (x, -y)$ & $-z = (-x, -y)$

$|z| = \sqrt{x^2+y^2}$, $|\bar{z}| = \sqrt{x^2+(-y)^2} = \sqrt{x^2+y^2}$

$|-z| = \sqrt{(-x)^2+(-y)^2} = \sqrt{x^2+y^2}$

Hence $|z| = |\bar{z}| = |-z|$

(iii) Let $z = (x, y)$, then $|z| = \sqrt{x^2+y^2}$

Now, $-\sqrt{x^2+y^2} \le x \le \sqrt{x^2+y^2}$, $-\sqrt{x^2+y^2} \le y \le \sqrt{x^2+y^2}$

$-|z| \le Re(z) \le |z|$, $-|z| \le Im(z) \le |z|$

(iv) Let $z = (x, y)$, $|z| = \sqrt{x^2+y^2}$

$\bar{z} = (x, -y)$

$z\bar{z} = (x, y) \cdot (x - y) = x^2 + y^2 = |z|^2$

$\therefore z\bar{z} = |z|^2$

(v) Let $z_1 = (x_1, y_1)$, $z_2 = (x_2, y_2)$ (13-24, 14+23)

$z_1 \cdot z_2 = (x_1, y_1) \cdot (x_2, y_2)$

$= (x_1x_2 - y_1y_2,\ x_1y_2 + x_2y_1)$

$|z_1 \cdot z_2| = |x_1x_2 - y_1y_2,\ x_1y_2 + x_2y_1|$

$= \sqrt{(x_1x_2 - y_1y_2)^2 + (x_1y_2 + x_2y_1)^2}$

$= \sqrt{x_1^2x_2^2 + y_1^2y_2^2 - \cancel{2x_1x_2y_1y_2} + x_1^2y_2^2 + x_2^2y_1^2 + \cancel{2x_1x_2y_1y_2}}$

$= \sqrt{x_1^2(x_2^2+y_2^2) + y_1^2(x_2^2+y_2^2)}$

$= \sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}$

$= \sqrt{x_1^2+y_1^2}\ \sqrt{x_2^2+y_2^2}$

$= |z_1| \cdot |z_2|$

$\therefore |z_1 \cdot z_2| = |z_1| \cdot |z_2|$

$\therefore$ Modulus of the product of two complex numbers is the product of their moduli

Generalisation:

$|z_1 \cdot z_2 \cdot z_3 \cdots z_n| = |z_1| \cdot |z_2| \cdot |z_3| \cdots |z_n|$

(vi) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$

$|z_1| = \left|\dfrac{z_1}{z_2} \cdot z_2\right| = \left|\dfrac{z_1}{z_2}\right| |z_2|$

$\dfrac{|z_1|}{|z_2|} = \left|\dfrac{z_1}{z_2}\right|$

$\therefore \left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$

(vii) $|z_1+z_2|^2 = (z_1+z_2)\overline{(z_1+z_2)}$ $\quad \because |z|^2 = z\bar{z}$, $\overline{z_1+z_2} = \bar{z}_1 + \bar{z}_2$

$= (z_1+z_2)(\bar{z}_1+\bar{z}_2)$

$= z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2$

$= |z_1|^2 + z_1\bar{z}_2 + z_2\bar{z}_1 + |z_2|^2$ $\quad \overline{(z_1\bar{z}_2)} = \bar{z}_1(\bar{\bar{z}}_2) = \bar{z}_1 z_2$

$= |z_1|^2 + |z_2|^2 + \{z_1\bar{z}_2 + \overline{(z_1\bar{z}_2)}\}$

$= |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z}_2)$ $\quad [\ z + \bar{z} = 2Re$

$|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z}_2)$

(viii) $|z_1 - z_2|^2 = (z_1+z_2)\overline{(z_1-z_2)}$ $\quad [\because z\bar{z} = |z|^2$

$= (z_1-z_2)(\bar{z}_1 - \bar{z}_2)$ $\quad [\because \overline{z_1-z_2} = \bar{z}_1 - \bar{z}_2$

$= z_1\bar{z}_1 + z_2\bar{z}_2 - z_1\bar{z}_2 - z_2\bar{z}_1$

$= |z_1|^2 + |z_2|^2 - (z_1\bar{z}_2 + \overline{z_1\bar{z}_2})$ $\quad \overline{(z_1\bar{z}_2)} = \bar{z}_1(\bar{\bar{z}}_2) = \bar{z}_1 \cdot z_2$

$= |z_1|^2 + |z_2|^2 - 2Re(z_1\bar{z}_2)$ $\quad [\because z+\bar{z} = 2Re$

$\therefore |z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2Re(z_1\bar{z}_2)$

(ix) Using (vii) & (viii)

$|z_1+z_2|^2 + |z_1-z_2|^2 = |z_1|^2 + |z_2|^2 + \cancel{2Re(z_1\bar{z}_2)} + |z_1|^2 + |z_2|^2 - \cancel{2Re(z_1\bar{z}_2)}$

$= 2(|z_1|^2 + |z_2|^2)$

Notes

Aliter

$|z_1+z_2|^2 + |z_1-z_2|^2 = (z_1+z_2)\overline{(z_1+z_2)} + (z_1-z_2)\overline{(z_1-z_2)}$

$= (z_1+z_2)(\bar{z}_1+\bar{z}_2) + (z_1-z_2)(\bar{z}_1-\bar{z}_2)$

$= z_1\bar{z}_1 + z_2\bar{z}_2 + \cancel{z_1\bar{z}_2} + \cancel{z_2\bar{z}_1} + z_1\bar{z}_1 + z_2\bar{z}_2 - \cancel{z_1\bar{z}_2} - \cancel{z_2\bar{z}_1}$

$= |z_1|^2 + |z_2|^2 + |z_1|^2 + |z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

The above result can be extended to
$|z_1+z_2+\cdots+z_n| \le |z_1|+|z_2|+\cdots+|z_n|$
$\left|\sum_{n=1}^{n} z_n\right| \le \sum_{n=1}^{n} |z_n|$

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Q. Prove that ~~that~~ the modulus of the sum of two complex numbers is less than or equal to the sum of their moduli

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i.e., $|z_1+z_2| \le |z_1|+|z_2|$

Proof: Let $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$
$\Rightarrow |z_1| = r_1$ $\Rightarrow |z_2| = r_2$

$z_1+z_2 = r_1 e^{i\theta_1} + r_2 e^{i\theta_2}$
$= r_1(\cos\theta_1 + i\sin\theta_1) + r_2(\cos\theta_2 + i\sin\theta_2)$
$= (r_1\cos\theta_1 + r_2\cos\theta_2) + i(r_1\sin\theta_1 + r_2\sin\theta_2)$

$|z_1+z_2|^2 = (r_1\cos\theta_1 + r_2\cos\theta_2)^2 + (r_1\sin\theta_1 + r_2\sin\theta_2)^2$
$= (r_1^2\cos^2\theta_1 + r_1^2\sin^2\theta_1) + (r_2^2\cos^2\theta_2 + r_2^2\sin^2\theta_2)$
$+ 2r_1r_2\cos\theta_1\cos\theta_2 + 2r_1r_2\sin\theta_1\cdot\sin\theta_2$

$= r_1^2 + r_2^2 + 2r_1r_2\cos(\theta_1-\theta_2)$
$\le r_1^2 + r_2^2 + 2r_1r_2$ $\quad [\because \cos(\theta_1-\theta_2) \le 1]$
$= (r_1+r_2)^2$
$|z_1+z_2| \le r_1 + r_2 = |z_1|+|z_2|$

$\therefore \boxed{|z_1+z_2| \le |z_1|+|z_2|}$ Hence Proved.

Q. Prove that the modulus of the difference of two complex number is greater ~~that~~ than or equal to difference of their moduli

i.e. $|z_1-z_2| \ge |z_1| - |z_2|$

Proof: Let $z_1 = r_1 e^{i\theta_1}$, $z_2 = r_2 e^{i\theta_2}$
$\Rightarrow |z_1| = r_1$ $\Rightarrow |z_2| = r_2$

$z_1 - z_2 = r_1 e^{i\theta_1} - r_2 e^{i\theta_2}$
$= r_1(\cos\theta_1 + i\sin\theta_1) - r_2(\cos\theta_2 + i\sin\theta_2)$

Notes
$z_1 - z_2 = (r_1\cos\theta_1 - r_2\cos\theta_2) + i(r_1\sin\theta_1 - r_2\sin\theta_2)$
$|z_1 - z_2| = \{(r_1\cos\theta_1 - r_2\cos\theta_2)^2 + (r_1\sin\theta_1 - r_2\sin\theta_2)^2\}^{1/2}$

12/05/20 U.C. Class/Sem/D... CO...

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2011 January

$|z_1-z_2|^2 = (r_1\cos\theta_1 - r_2\cos\theta_2)^2 + (r_1\sin\theta_1 - r_2\sin\theta_2)^2$

25 TUE

$|z_1-z_2|^2 = (r_1\cos\theta_1 - r_2\cos\theta_2)^2 + (r_1\sin\theta_1 - r_2\sin\theta_2)^2$
$= r_1^2\cos^2\theta_1 + r_2^2\cos^2\theta_2 - 2r_1r_2\cos\theta_1\cdot\cos\theta_2$
$+ r_1^2\sin^2\theta_1 + r_2^2\sin^2\theta_2 - 2r_1r_2\sin\theta_1\cdot\sin\theta_2$
$= r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1-\theta_2)$
$\ge r_1^2 + r_2^2 - 2r_1r_2$ $\quad \cos(\theta_1-\theta_2) \le 1$
$= (r_1-r_2)^2$ $\quad -\cos(\theta_1-\theta_2) \ge$
$\therefore |z_1-z_2| \ge r_1 - r_2 = |z_1| - |z_2|$

$\therefore \boxed{|z_1-z_2| \ge |z_1| - |z_2|}$ Hence Proved

(x) $|z^n| = |z.z.z\cdots n\text{ times}| = |z||z|\cdot|z|\cdots n\text{ times} = |z|^n$

Properties of Argument of Two Complex num

26 WED

(1) The argument of the product of two complex number is equal to the sum of their arguments.

i.e, $\arg(z_1\cdot z_2) = \arg(z_1) + \arg(z_2)$

Proof: Let $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$
Here $\arg(z_1) = \theta_1$ and $\arg(z_2) = \theta_2$
$z_1\cdot z_2 = r_1 e^{i\theta_1}\cdot r_2 e^{i\theta_2} = r_1 r_2 e^{i(\theta_1+\theta_2)}$
$\therefore \arg(z_1 z_2) = \theta_1+\theta_2 = \arg(z_1) + \arg(z_2)$ Proved

(ii) The argument of the quotient of two complex numbers is equal to the difference of their arguments.
i.e, $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

Notes

※ The Conjugate of the sum/of two ... the sum/ of their conjugates difference

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Proof: Let $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$

27 THU

Then $\arg(z_1) = \theta_1$ and $\arg(z_2) = \theta_2$

Now, $$z_1/z_2 = \frac{r_1 e^{i\theta_1}}{r_2 e^{i\theta_2}} = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2)}$$

$$\therefore \arg(z_1/z_2) = \theta_1 - \theta_2 = \arg(z_1) - \arg(z_2)$$

$$\therefore \arg(z_1/z_2) = \arg(z_1) - \arg(z_2)$$ Proved.

(iii) $\arg(z^n) = \arg(z.z.z\ldots n\text{ times}) = \arg z + \arg z + \arg z + \cdots n\text{ times} = n\arg(z)$

Properties of Conjugates –

If $z_1, z_2, z_3 \in C$, then

(i) $\overline{(\overline{z})} = z$

Let $z = (x, y)$

$\Rightarrow \overline{z} = (x, -y)$

$\Rightarrow \overline{(\overline{z})} = (x, y) = z$

28 FRI

(ii) $z + \overline{z} = (x, y) + (x, -y) = (x+x, y-y) = (2x, 0)$

$\Rightarrow z + \overline{z} = 2x = 2\,\text{Re}(z) \Rightarrow \text{Re}(z) = \frac{z+\overline{z}}{2}$

If $z + \overline{z} = 0 \Leftrightarrow 2x = 0 \Leftrightarrow x = 0 \Leftrightarrow \text{Re}(z) = 0 \Leftrightarrow z$ is purely imaginary

(iii) $z - \overline{z} = (x, y) - (x, -y) = (x-x, y+y) = (0, 2y)$

$z - \overline{z} = 2i\,\text{Im}(z) \Rightarrow \text{Im}(z) = \frac{z-\overline{z}}{2i}$

If $z - \overline{z} = 0 \Rightarrow 2i\,\text{Im}(z) = 0 \Rightarrow \text{Im}(z) = 0 \Rightarrow y = 0 \Rightarrow z$ is purely real.

(iv) $z\overline{z} = (x, y).(x, -y) = x^2 + y^2 = \sqrt{\{\text{Re}(z)\}^2 + \{\text{Im}(z)\}^2}$

(v) $\overline{z_1 + z_2} = \overline{(x_1, y_1) + (x_2, y_2)} = \overline{(x_1 + x_2, y_1 + y_2)} = (x_1 + x_2), -(y_1 + y_2)$

※ $= (x_1, -y_1) + (x_2, -y_2) = \overline{(x_1, y_1)} + \overline{(x_2, y_2)} = \overline{z_1} + \overline{z_2}$

(vi) $\overline{z_1 - z_2} = \overline{(x_1, y_1) - (x_2, y_2)} = \overline{(x_1 - x_2, y_1 - y_2)} = (x_1 - x_2, -(y_1 - y_2))$

Notes

※ $= (x_1, -y_1) - (x_2, -y_2) = \overline{(x_1, y_1)} - \overline{(x_2, y_2)}$

$= \overline{z_1} - \overline{z_2}$

P.T.O →

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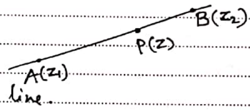
Equation of a straight line

29 SAT

To find the equation of a straight line joining two given points.

Let $A(z_1)$ and $B(z_2)$ be two points of joining line AB



Let $P(z)$ be any point on this line.

$$\arg\left(\frac{z - z_1}{z_1 - z_2}\right) = 0 \text{ or } \pi$$

$\Rightarrow \frac{z - z_1}{z_1 - z_2}$ is purely real

$$\Rightarrow \text{Im}\left(\frac{z - z_1}{z_1 - z_2}\right) = 0$$

$$\frac{1}{2i}\left[\left(\frac{z - z_1}{z_1 - z_2}\right) - \overline{\left(\frac{z - z_1}{z_1 - z_2}\right)}\right] = 0 \quad \text{as } 2i\,\text{Im}(z) = z - \overline{z}$$

30 SUN

$$\frac{z - z_1}{z_1 - z_2} = \overline{\left(\frac{z - z_1}{z_1 - z_2}\right)} = \frac{\overline{z - z_1}}{\overline{z_1 - z_2}} = \frac{\overline{z} - \overline{z_1}}{\overline{z_1} - \overline{z_2}}$$

$\because \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$, $\overline{z - z_1} = \overline{z} - \overline{z_1}$

$$\frac{z - z_1}{z_1 - z_2} = \frac{\overline{z} - \overline{z_1}}{\overline{z_1} - \overline{z_2}}$$

$$(z - z_1)(\overline{z_1} - \overline{z_2}) = (z_1 - z_2)(\overline{z} - \overline{z_1})$$

$$\Rightarrow z(\overline{z_1} - \overline{z_2}) - \overline{z}(z_1 - z_2) \cancel{-z_1\overline{z_1}} + z_1\overline{z_2} \cancel{+z_1\overline{z_1}} - \overline{z_1}z_2 = 0$$

$$\therefore z(\overline{z_1} - \overline{z_2}) - \overline{z}(z_1 - z_2) + (z_1\overline{z_2} - \overline{z_1}z_2) = 0 \quad \text{— (1)}$$

Which is required equation of the line.

General Equation of a Line –

Multiplying (1) by i, we get

$$iz(\overline{z_1} - \overline{z_2}) - i\overline{z}(z_1 - z_2) + i(z_1\overline{z_2} - \overline{z_1}z_2) = 0 \quad \text{–(2)}$$

Let $-i(z_1 - z_2) = a$

Notes

So $\overline{a} = i(\overline{z_1} - \overline{z_2})$

* Also $z_2\bar{z}_1$ is Conjugate of $z_1\bar{z}_2$ and so their difference $z_1\bar{z}_2 - z_2\bar{z}_1$ is purely imaginary
Hence $i(z_1\bar{z}_2 - z_2\bar{z}_1)$ is purely real say c

$\overline{z_1 \cdot \bar{z}_2} = \bar{z}_1 \cdot \bar{\bar{z}}_2 = \bar{z}_1 \cdot z_2$

② $\therefore (\overline{z_1 \cdot \bar{z}_2})$

$(z_1 \cdot \bar{z}_2 - \bar{z}_1 \cdot z_2)$ = purely imaginary number & so

$i(z_1\bar{z}_2 - \bar{z}_1 z_2)$ = purely real number $= c$ (say)

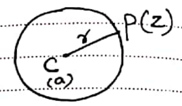
Now, from (2)

$\boxed{\bar{a}z + a\bar{z} + c = 0}$ where $a \neq 0$ and c is real

which is required general equation of a straight line.

Equation of a Circle

To find the equation of a circle with centre a and radius r



Let us consider a circle with centre $C(a)$ and radius r where a is any complex number. Let $P(z)$ be any point on the circle

Then,

length of ~~the circl~~ CP = radius of the circle

$|z-a| = r$

$|z-a|^2 = r^2$

$\Rightarrow (z-a)(\overline{z-a}) = r^2$

$\Rightarrow (z-a)(\bar{z}-\bar{a}) = r^2$

$\Rightarrow z\bar{z} - \bar{a}z - a\bar{z} + a\bar{a} = r^2$

$\Rightarrow z\bar{z} - \bar{a}z - a\bar{z} + (|a|^2 - r^2) = 0$ = real number

$\Rightarrow \boxed{z\bar{z} - \bar{a}z - a\bar{z} + (|a|^2 - r^2) = 0}$ — (1)

which is req. eqn of the circle

* $(z+b)(\overline{z+b}) = b\bar{b} - c$

$|z+b|^2 = b\bar{b} - c = (\sqrt{b\bar{b}-c})^2$

$|z+b| = \sqrt{b\bar{b}-c}$

Comparing this with (8), $|z-a| = r$, we conclude that it represents a circle whose centre is $-b$ and radius $\sqrt{b\bar{b}-c}$ provided $(b\bar{b}-c) > 0$

General Equation of the Circle.

Let $-a = b$, then $\overline{-a} = \bar{b}$ and $|a|^2 - r^2 = c$ = real no

$\Rightarrow -(\bar{a}) = \bar{b}$ $\quad (\overline{-z} = -(\bar{z}))$

$\therefore$ from (1)

$\boxed{z\bar{z} + \bar{b}z + b\bar{z} + c = 0}$

where c is real and b is Complex ~~number~~ Constant

This is called General equation of the circle.

Conversely We now show that $z\bar{z} + b\bar{z} + \bar{b}z + c = 0$ will represent a circle

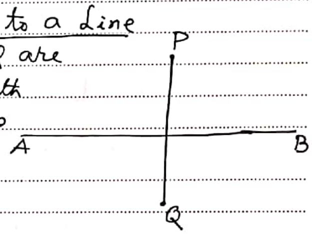
~~∴~~ The above eqn can be written as

$z\bar{z} + b\bar{z} + \bar{b}z + b\bar{b} = b\bar{b} - c$ [adding $b\bar{b}$ to both sides

$(z+b)(\bar{z}+\bar{b}) = b\bar{b} - c$ *

Inverse points with respect to a Line

Two points P and Q are said to the inverse points with respect to the line AB if Q is the image of P in AB. i.e. if the line AB is the right bisector of PQ.



Inverse points with respect to a circle:

Two points P and Q are said to be the inverse points with respect to a circle S if they are Collinear with the centre and if their distances from the centre is equal to r^2 where r is the radius of the circle i.e., $CP \cdot CQ = r^2$